J. Fluid Mech. (1968), *vol.* 33, *part* 2, *pp.* 283–292
Printed in Great Britain

Effect of streamwise wall curvature on heat transfer in a turbulent boundary layer

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(Received 10 October 1967 and in revised form 9 January 1968)

Experiments described here show that the rate of heat transfer in a supersonic, turbulent, boundary layer on a concave wall is increased by the streamwise curvature of the wall. For the arrangement investigated, the pressure is kept constant along the wall, and the increase of about 20 % is therefore only due to the wall curvature. For a convex wall, a comparable decrease was found, also with constant pressure along the wall.

It is likely that this change in heat-transfer rate is mainly due to an increase or decrease of turbulent mixing by the effect of the curvature. The increase on a concave wall can in principle also be explained by large-scale vortices with axes in the flow direction (Görtler vortices). However, disturbances of this type cannot explain the decrease observed on the convex wall. They cannot therefore be the only cause.

A simple criterion indicating the effect of curvature on the turbulent motion normal to the wall is given. It is derived from an inviscid-flow analysis. The criterion shows that, for most Mach numbers and wall temperatures of practical interest, the change in mixing depends mainly on the velocity gradient normal to the wall. For high supersonic and hypersonic Mach numbers, however, there is a layer near the outer edge of the boundary layer where the change depends mainly on the temperature (density) gradient.

1. Introduction

Boundary-layer theory is based on the assumption that the pressure gradient normal to the wall can be neglected. If the wall has a streamwise curvature the assumption is not always valid. In incompressible flow, a boundary layer on a concave wall will produce a moment-of-momentum distribution that is unstable in most parts of the boundary layer. A particle displaced in the direction normal to the wall will not return to its original streamline. This instability has two effects. On concave walls it will increase the intensity of the turbulent mixing (see, for example, Margolis & Lumley 1965), and it can produce large-scale vortices with axes in the flow direction (Görtler vortices), as predicted and described by Görtler (1959), Sandmayr (1966) and Tani (1962).

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Over a large part of a supersonic boundary layer, the density decreases towards the wall. On a concave wall this density distribution is another source of instability. It was shown by Görtler (1959) that it is closely related to the instability associated with the velocity distribution found in incompressible flow.

Few investigations of the effect of wall curvature at supersonic speeds have been published. Michel (1960) presents velocity profiles measured on curved walls at supersonic speeds. They show an appreciable effect of the curvature. Rotta's (1966) theoretical investigation gives velocity profiles close to the wall which also indicate an effect of the wall curvature. Both investigations suggest that the turbulent mixing is increased on a concave wall, but they do not give a clear indication of the magnitude of the effect.

Turbulent boundary layers on curved walls are found in many cases, as, for example, on the forward part of a blunt body, on isentropic-compression air intakes, in the expansion at the separation point of a base-flow problem, etc. In all these cases, however, only a combination of this effect with the effect of the pressure gradient in the flow direction is usually observed. No measurements that show the influence of the curvature alone on heat transfer are known to the author. The aim of the present investigation is to determine the magnitude of this effect. The effect on skin friction should be similar, but is much more difficult to measure.

2. Experimental results

The present investigation was conducted in a supersonic blow-down tunnel at the Aeronautical Research Institute of Sweden (FFA). The free-stream Mach number was 2·5 and the Reynolds number based on total model length was about $4{\cdot}5 \times 10^6$. The pressure gradient along the curved walls tested could be eliminated by fitting suitably shaped bodies opposite the test surface. The shape of these bodies was calculated by the method of characteristics with a correction for the displacement effect of the boundary layer. Heat-transfer rates and pressure distributions were measured for the configurations shown in figure 1.

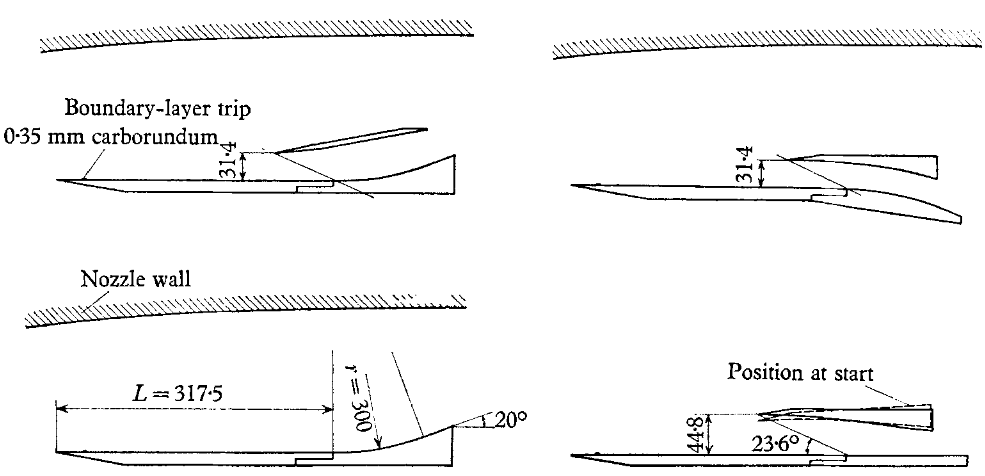

FIGURE 1. Configurations tested, tunnel width = 234 mm (all dimensions in mm).

The boundary layer was tripped by 0·35 mm carborundum grains at the locations shown.

Heat transfer to the surface was measured by a transient method. With this method, a thin-walled model is pre-cooled and then suddenly exposed to the flow field. An element of the thin wall is used as calorimeter. The relation between the heat added to the element and the increase of temperature is used to determine the heat-transfer rate.

In the present case the test surface consisted of a 1 mm thick stainless-steel sheet. Before a run was made, themodel was cooledwith a layer of solid carbon dioxide to about 195 °K or heated with warm air to about 330 °K. The tunnel was then started (with a starting time less than 0·5 sec) and the wall temperatures were measured by iron–constantan thermocouples. By using an energy balance for the wall element, the measured rate of temperature increase was related to the heat-transfer rate. Full details of the experimental arrangement and of the data reduction are given by Thomann (1967).

The measured pressure distribution for a flat plate is shown in figure 2. Distributions for the curved surfaces and the flat plate with a pressure gradient are shown in figure 3. Here the co-ordinate x is measured from the leading edge of the model and is referred to L, where L is the length shown in figure 1. The curvature or pressure gradient starts at $x/L = 1$. Pressures are referred to the free-stream stagnation pressure. As can be seen, the distributions agree reasonably well with ideal-flow theory (inviscid two-dimensional flow with a free-stream Mach number of 2·5).

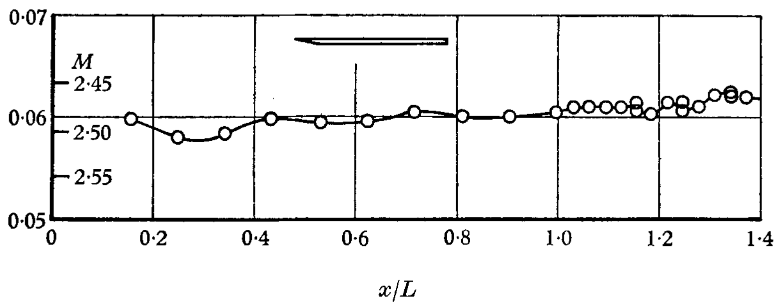

FIGURE 2. Pressure distribution for a flat plate.

In figures 4–7 the heat-transfer results are presented in terms of the Stanton number, which is defined as

$$St = \frac{q}{\rho_e u_e c_p (T_r - T_w)}. \tag{1}$$

Here, q is the rate of heat transfer to the wall, c_p the specific heat of air and T_w the wall temperature. The recovery temperature T_r was calculated assuming a recovery factor of 0·88, and the density ρ_e and the velocity u_e were calculated from the measured stagnation conditions and wall pressure, assuming isentropic flow.

Figure 4 shows the effect of the boundary-layer trip on the heat-transfer results for the flat plate. Without the trip, transition from laminar to turbulent flow takes place in the range $0{\cdot}4 \times 10^6 \leqslant Re \leqslant 1{\cdot}2 \times 10^6$. With a small trip (0·22 mm), transition was essentially terminated at $Re = 0{\cdot}4 \times 10^6$, and, with a larger trip (0·35 mm), turbulent flow was observed at about $Re = 0{\cdot}2 \times 10^6$. The results are compared with Eckert's (1955) reference-temperature method and with Walz's (1966) Rechenverfahren II. Excellent agreement with Eckert's method is found except, of course, near transition. For the tests on the shapes shown in figure 1, the larger trip was used.

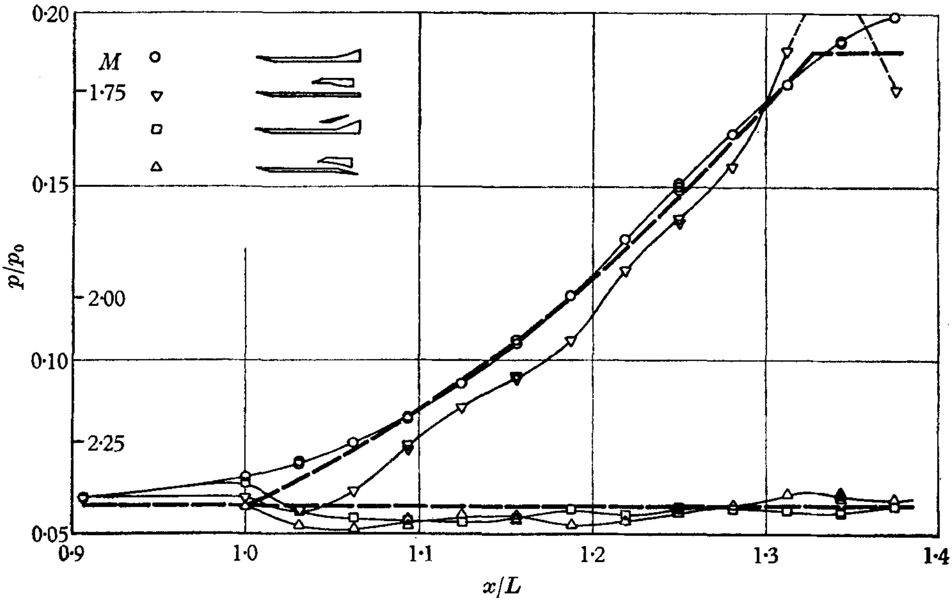

FIGURE 3. Pressure distribution for curved walls; — —, ideal-flow analysis.

In figure 5 the effect of wall curvature on heat transfer is shown. In all three cases the pressure is essentially constant along the wall. It is seen that a concave wall increases the heat-transfer rate by about 20 % and that the convex wall decreases it by about the same amount. The turbulent mixing inside the boundary layer is changed. On the concave wall it is increased and on the convex wall it is decreased. Apparently, this effect also changes the heat transferred to the wall. This change is gradual and takes about 10–15 boundary-layer thicknesses to develop.

If the wall is heated, the difference between the free-stream and the wall density increases. This affects the stability of laminar boundary layers and may influence mixing in turbulent layers. In order to determine the influence, the experiments were repeated with a heated wall. The results are shown in figure 6. Again, the effect of the curvature is clearly seen but the difference between the two curves did not increase.

In figure 7 the effect of an axial pressure gradient is shown. The pressure gradient along the curved wall was in this case no longer compensated. On the flat plate a pressure distribution similar to that on the curved wall was induced

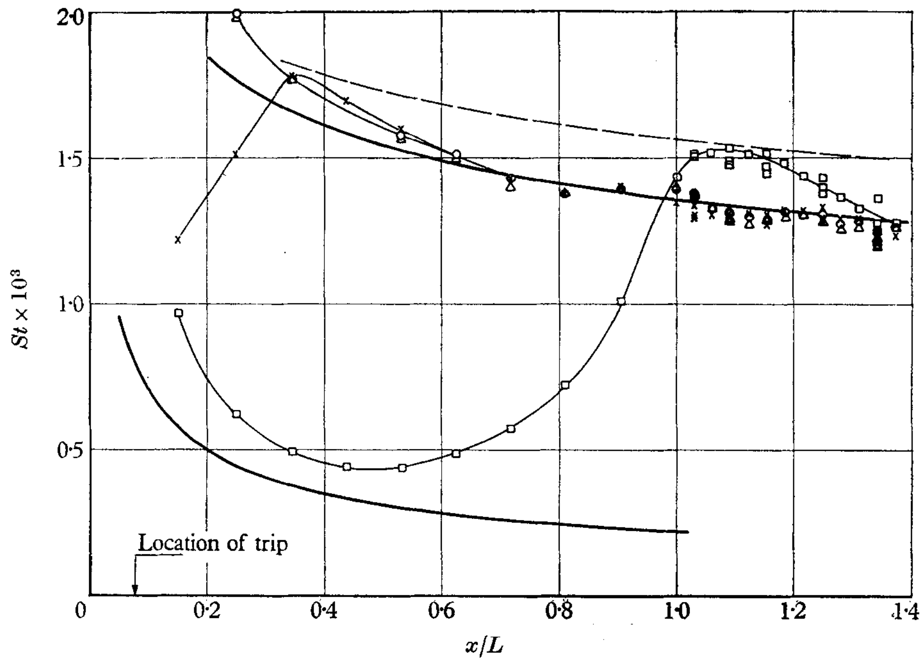

FIGURE 4. Heat transfer to a flat plate; $M = 2{\cdot}5$, $T_0 = 290\,°K$, $T_w = 212\,°K$, $Re_L = 3{\cdot}21 \times 10^6$. □, no trip; ×, 0·2 mm trip; △, ○, 0·35 mm trip; ——, theory (Eckert 1955); — — —, theory (Walz 1966).

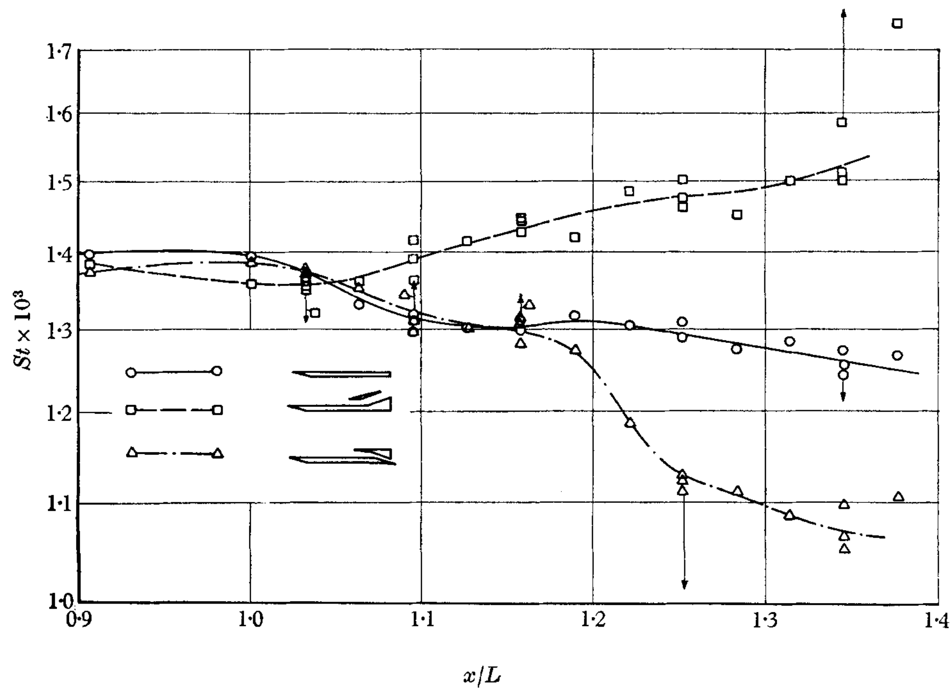

FIGURE 5. Effect of wall curvature on heat transfer, for constant pressure along a cooled wall; $T_0 = 290\,°K$, $T_w = 212{\cdot}5\,°K$. The wall curvature starts at $x/L = 1$.

by a body above the test surface (see figure 1). Thus a curved and a flat wall, both with a pressure gradient in the flow direction, were compared. As in the case of constant pressure along the wall the concave curvature of the wall increases the heat transfer by about 20 %.

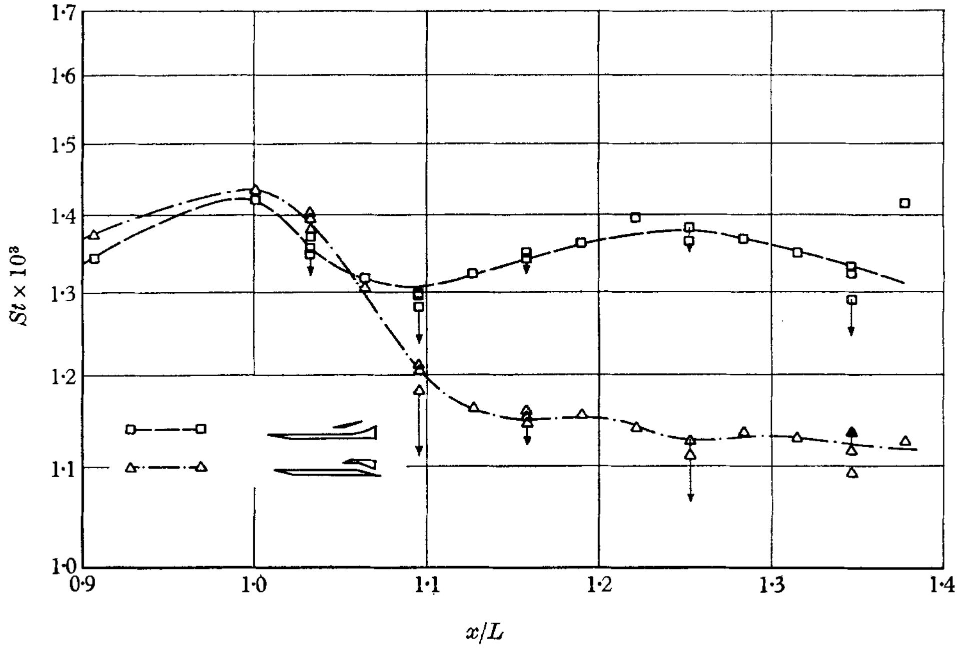

FIGURE 6. Effect of wall curvature on heat transfer for constant pressure along a heated wall, T_0 = 290 °K, T_w = 322·5 °K.

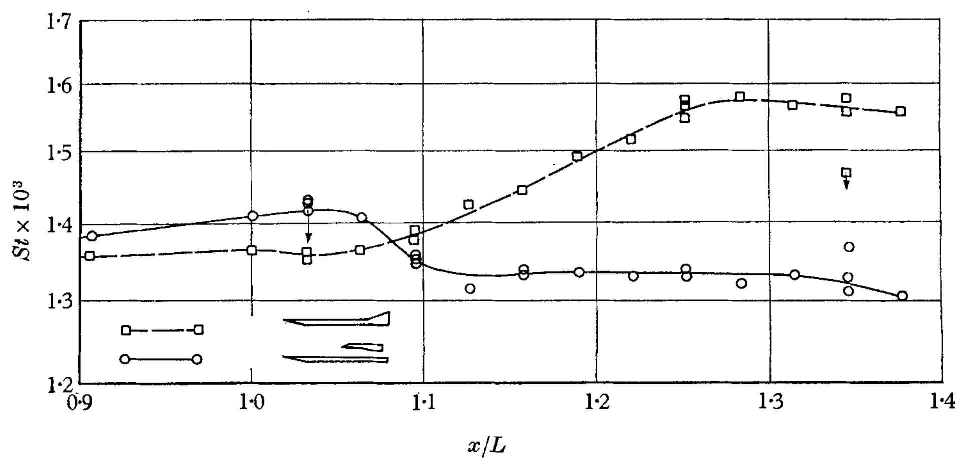

FIGURE 7. Effect of wall curvature on heat transfer with an adverse pressure gradient; T_0 = 290 °K, T_w = 212·5 °K.

3. Acceleration of a displaced fluid particle

An indication of the effect of the wall curvature on the turbulent mixing is found if variations of all quantities in the flow direction are neglected and if it is assumed that a fluid particle that is displaced normal to a streamline preserves its moment of momentum and its entropy. Viscous effects are thus neglected. If a fluid particle that is in equilibrium on streamline 1 in figure 8 is displaced to streamline 2, it will be subject to a radial acceleration

$$\frac{d\bar{v}}{dt} = -\frac{\bar{u}^2}{r} - \frac{1}{\bar{\rho}}\frac{dp}{dy}, \tag{2}$$

where ρ is the density, p the pressure, r the radius of curvature of the wall and u

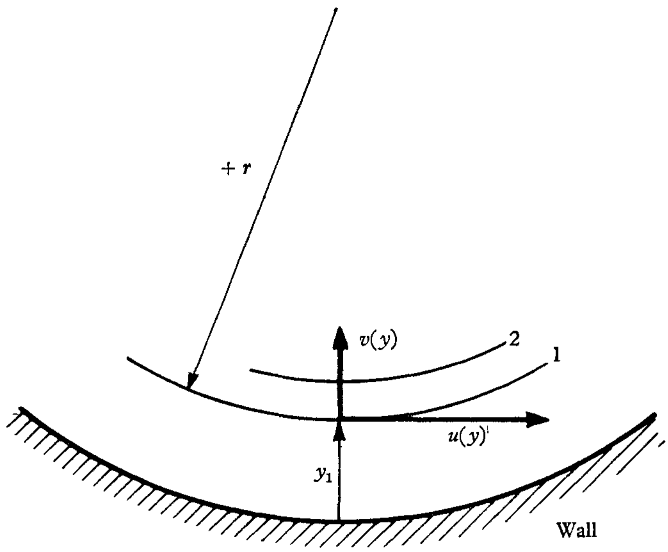

FIGURE 8. Flow along a curved wall.

and v are the velocity components parallel and normal to the wall. The symbols marked with a bar refer to the particle in the new position on streamline 2. The pressure gradient in equation (2) is implied by the flow surrounding the particle. It is given by

$$\frac{dp}{dy} = -\frac{\rho u^2}{r}. \tag{3}$$

If all quantities in (2) and (3) are described by Taylor series and if only terms linear in $y_2 - y_1$ are retained, the following result is obtained:

$$\frac{d\bar{v}}{dt} = \frac{u^2}{r}\left\{\frac{2\delta}{u}\frac{du}{dy} - \frac{2\delta}{r} - (\gamma-1)M^2\frac{\delta}{r} - \frac{\delta}{T}\frac{dT}{dy}\right\}\frac{y_2 - y_1}{\delta}. \tag{4}$$

Here, M is the local Mach number, δ a typical length characterizing the boundary layer and $y_2 - y_1$ the displacement of the particle normal to the wall. The temperature gradient dT/dy and the velocity gradient du/dy are given by the undisturbed flow field.

If $d\bar{v}/dt$ and $y_2 - y_1$ have opposite sign, the particle will return towards streamline 1 and the curvature of the wall has a stabilizing effect. If they have the same sign, the effect is de-stabilizing. For incompressible flow it was shown by Margolis & Lumley (1965) that a de-stabilizing tendency will increase the turbulent exchange across streamlines and thus the transport of momentum. It is very likely that the same is true also for a compressible boundary layer.

The first two terms in the braces of equation (4) are contributions from the velocity changes of the surroundings and of the particle respectively. They are the same as for incompressible flow. The third term takes into account the change of density due to the pressure gradient, and the last term gives the corresponding effect of the temperature gradient.

It is illustrative to apply equation (4) to a typical case investigated in the experiments. In order to simplify the calculations, the following approximations were used: (i) a linear relation is assumed to exist between the velocity u and the local stagnation temperature ($Pr = 1$), (ii) in the outer part of the boundary layer, the velocity profile is approximated by $u/u_e = (y/\delta)^{\frac{1}{7}}$, (iii) near the wall the velocity was approximated by $u = \tau y/\mu_w$, where τ is the shear stress at the wall, and μ_w the viscosity there.

The calculations were based on the following figures, typical for the present experiments: $M_e = 2{\cdot}48$, $Re_L = 3{\cdot}2 \times 10^6$, $T_w = 212{\cdot}5\,°\text{K}$ and $322{\cdot}5\,°\text{K}$, $T_e = 130\,°\text{K}$, $r = 0{\cdot}3\,\text{m}$, $\delta = 6{\cdot}3 \times 10^{-3}\,\text{m}$, where the subscript e is used to denote the free-stream state. The boundary-layer thickness δ was calculated from the momentum and displacement thickness which had been computed by Walz's (1966) method. The results are shown in figure 9. In this figure negative contributions have a de-stabilizing effect. It is seen that essentially the whole boundary layer is unstable. For incompressible flow the velocity gradient is the only de-stabilizing influence, and this effect is still dominant at $M_e = 2{\cdot}5$. In most parts of the boundary layer the effect of the temperature gradient is much smaller, especially in the case of a cooled wall. It is therefore likely that this explains why the results for cooled and heated walls shown in figures 5 and 6 are so similar.

It is also interesting to note that the instability is strongest near the edge of the laminar sublayer, where the turbulence production is a maximum. The effects of the Coriolis acceleration and of the isentropic change of state of the particle are negligible for the present conditions.

In figure 10 the effect of the free-steam Mach number is shown. This figure is based on the assumption that the ratio δ/r and the relation $u/u_e = f(y/\delta)$ are independent of the Mach number. With increasing Mach number and wall temperature, an increasing part near the outer edge of the boundary layer is dominated by the de-stabilizing effect of the temperature gradient. Closer to the wall the effect of the temperature gradient is strongly dependent on the wall temperature, but for all practical cases it is dominated by the de-stabilizing effect of the velocity distribution.

It must be kept in mind that figures 9 and 10 are based on an idealized picture of the flow. Viscous effects will decrease the difference in velocity and temperature between the displaced fluid particle and its new surroundings, thus decreasing the de-stabilizing effect of the exchange. Furthermore, the driving force shown in figures 9 and 10 must be compared with the retarding forces induced by the motion itself before conclusions can be drawn about the increase of the turbulent mixing.

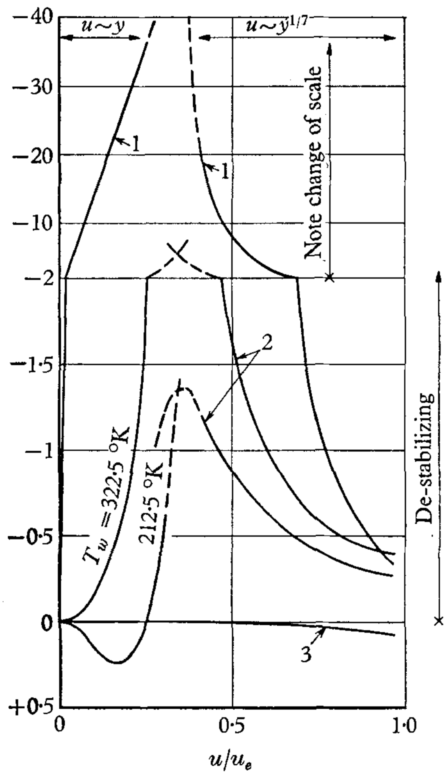

FIGURE 9

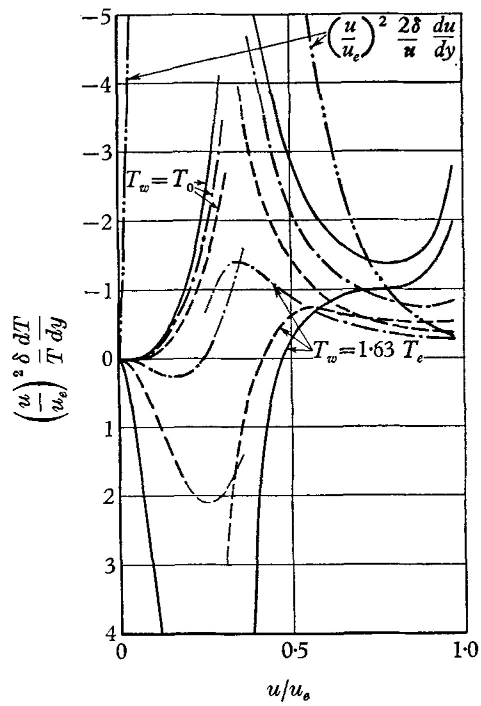

FIGURE 10

FIGURE 9. Contributions to the instability of the boundary layer; $M = 2{\cdot}5$, $T_0 = 290\,°\mathrm{K}$. Curve 1, the effect of $(u/u_e)^2(2\delta/u)du/dy$; curve 2, of $(u/u_e)^2(\delta/T)\,dT/dy$; curve 3, of $(u/u_e)^2\{2+(\gamma-1)M^2\}\delta/r$.

FIGURE 10. Effect of Mach number on the stability of the boundary layer. ——, $M = 10$; — — —, $M = 4$; — · —, $M = 2{\cdot}5$.

This research has been sponsored by the Royal Swedish Air Force Board. The author is indebted to Mr R. Hellström who conducted most of the experiments with great skill and care.

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